## Lecture 12-minimum spanning trees

1. What is a minimum spanning tree (MST)?

a) A tree with the least number of edges

b) A subgraph that is both a tree and includes all vertices with minimum total edge weight

c) The shortest path between any two vertices

d) A tree with the maximum number of edges

Answer: b

2. Which of the following is NOT a characteristic of a spanning tree?

a) Connected

b) Acyclic

c) Includes all vertices

d) Contains loops

Answer: d

3. What is the main goal when finding a minimum spanning tree?

a) Maximize the number of edges

b) Minimize the number of vertices

c) Find the shortest path between two vertices

d) Minimize the total weight of edges

Answer: d

4. Which of these is an application of minimum spanning trees?

a) Social network analysis

b) Laying cable in a new neighborhood

c) Sorting algorithms

d) Database indexing

Answer: b

5. What is the first step in Kruskal's algorithm?

a) Start with a random vertex

b) Consider edges in descending order of weight

c) Consider edges in ascending order of weight

d) Add all edges to the tree

Answer: c

6. In Kruskal's algorithm, when is an edge NOT added to the tree?

a) When it creates a cycle

b) When it's the heaviest edge

c) When it's already in the tree

d) When it connects two different components

Answer: a

7. What data structure is commonly used in the implementation of Kruskal's algorithm?

a) Stack

b) Queue

c) Union-find

d) Hash table

Answer: c

8. What is the starting point for Prim's algorithm?

a) The heaviest edge

b) A random vertex

c) Usually vertex 0

d) The lightest edge

Answer: c

9. In Prim's algorithm, how are edges selected to be added to the tree?

a) Randomly

b) Based on their endpoints

c) The minimum weight edge with exactly one endpoint in the tree

d) The maximum weight edge with both endpoints outside the tree

Answer: c

10. How many edges does Prim's algorithm add to create the MST?

a) V

b) V - 1

c) E

d) E - 1

Answer: b

11. What is a key similarity between Dijkstra's algorithm and Prim's algorithm?

a) They both find the shortest path

b) They both use a greedy approach

c) They both require negative edge weights

d) They both produce a maximum spanning tree

Answer: b

12. What is a fundamental difference between Dijkstra's and Prim's algorithms?

a) Dijkstra's uses a priority queue, while Prim's doesn't

b) Prim's works on undirected graphs, while Dijkstra's doesn't

c) Dijkstra's finds shortest paths, while Prim's constructs a minimum spanning tree

d) Prim's is faster than Dijkstra's

Answer: c

13. In Dijkstra's algorithm, how is the distance to a vertex updated?

a) By adding the weight of the new edge

b) By subtracting the weight of the new edge

c) By relaxing to the sum of the edge weight plus the distance to the previous vertex if smaller

d) By choosing the maximum of the current distance and the new path distance

Answer: c

14. In Prim's algorithm, how is the next vertex chosen?

a) The vertex with the highest degree

b) The vertex with the lowest degree

c) The vertex with the minimum weight edge connecting it to the MST

d) The vertex farthest from the starting point

Answer: c

15. What is the time complexity of Kruskal's algorithm with sorting?

a) O(V log V)

b) O(E log E)

c) O(V^2)

d) O(E^2)

Answer: b

16. What is the time complexity of Prim's algorithm using a priority queue?

a) O(V log V)

b) O(E log V)

c) O(V^2)

d) O(E^2)

Answer: b

17. Which algorithm is generally faster for sparse graphs?

a) Kruskal's

b) Prim's

c) They are always equally fast

d) It depends on the implementation

Answer: a

18. What property of a graph ensures that a minimum spanning tree exists?

a) The graph must be directed

b) The graph must be connected

c) The graph must have negative edge weights

d) The graph must be acyclic

Answer: b

19. In the context of MST, what does "greedy" mean?

a) The algorithm always chooses the largest possible solution

b) The algorithm makes the locally optimal choice at each step

c) The algorithm requires the most computational resources

d) The algorithm only works on complete graphs

Answer: b

20. Which of the following is NOT a typical application of minimum spanning trees?

a) Network design

b) Cluster analysis

c) Sorting algorithms

d) Approximation algorithms for NP-hard problems

Answer: c